Engineer's Notebook

aids for the structural engineer's toolbox

Slender Reinforced Concrete Walls

The Merging of Design Philosophies

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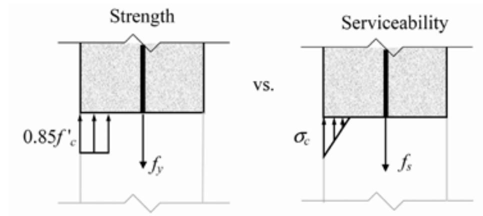
Limit states design – also known as ultimate strength design or load and resistance factor design (LRFD) – is largely supplanting the traditional methods of allowable stress design for most structural materials. Perhaps you are seasoned enough to remember the days when working stress design of reinforced concrete was the norm, and limit states design was a fairly new concept. However, for most of us, limit states design has always been the norm, though some remnants of working stress design have endured. For instance, when the deflection of a concrete beam comes into question, we revert to methods drawn from the working stress design theory.

Why is this necessary? The answer is simple and grounded in the basic theory of limit states design. When evaluating a "strength" limit state – e.g., flexural design of a reinforced concrete beam – we are considering the *ultimate* failure state of the member. We know that the likelihood of a sufficiently designed member ever reaching this failure state is extremely small and that, if it does happen, the member will fail "in a certain way" so that its behavior is controlled, manipulated and even predictable; ductile is another descriptor that comes to mind. Calculation of deflections at the ultimate limit state is not pragmatic and probably not realistic, since even the best tools at our disposal cannot reliably predict deflections (strains) that occur in concrete as it is being crushed.

More to the point, this is such an extreme condition that it does not reflect a serviceability (deflection) limit state. We are simply satisfied in assuming that concrete begins to crush at a compressive strain of 0.003 and that we can approximate its average stress over the whole of the compression zone as 85% of f'_c. This is why we must revert to working stress methods to check deflections – we are looking to calculate the anticipated deflections under realistic (un-factored) loads that probably do not push the member anywhere near yielding of steel, let alone (if we have designed it correctly) crushing of concrete.

The need for both limit states design methods and working stress design methods in reinforced concrete is perhaps most evident if we look at slender walls as addressed by the American Concrete Institute's ACI 318-11, section 14.8. The slenderness of the element not only exacerbates bending loads due to the P-delta effect, but also potentially causes appreciable out-of-plane deflections. The first part of this section is, in essence, a check to ensure that factored flexural loads (M_u) do not exceed flexural capacities (ϕM_n) as expressed in equation 14-3. The ϕM_n calculation is trivial, but the factored moment (M_u) becomes a little more challenging as it must account for the P-delta effect. Required within this approach is the out-of-plane deflection (Δ_u) at the wall mid-height for the load combination in question.



Assumed Strength vs. Service Stress Profiles

Fortunately, ACI 318-11 prescribes the equations for this, which include several variables for which calculations are relatively straightforward. Consider Δ_u, the theoretical displacement of the wall at mid-height under the ultimate load (M_u); note that this is not the displacement considering realistic service loads. Solving equation 14-5 for Δ_u and equation 14-4 for M_u typically requires at least a few iterations; for example, the first attempt could assume zero deflection. Conveniently, equation 14-6 provides direct calculation of M_u, implicitly accounting for the P-delta effect. Interestingly, this approach uses cracked section properties, a rational (and probably conservative) presumption since we are applying *ultimate* loads that will likely produce stresses beyond the concrete modulus of rupture. Note that ACI 318-11 section 14.8.3 does not prescribe a maximum limit for this (ultimate) displacement.

Moving on, ACI 318-11 section 14.8.4 prescribes a maximum out-of-plane deflection Δ_s due to *service* loads of $l_c/150$. The ACI procedure for this calculation accounts for several factors. Chief among them is whether the anticipated rupture (tensile) stress of the concrete has been exceeded. Obviously, if the rupture stress is not surpassed, the wall effectively acts as a homogeneous material and deflections will probably not be excessive. On the other hand, if the rupture stress is surpassed, then the wall cracks, the reinforcement tensile mechanism mobilizes, and cracked section properties (e.g., moment of inertia) become significantly altered such that the out-of-plane deflections become relatively high. Whether the concrete remains uncracked is thus an important serviceability issue – one that we do not even consider when we look at strength design.

ACI 318 section 14.8.4 prescribes an 'iteration of deflections' approach for determining the balanced condition of applied loads, P-delta effects and out-of-plane deflection. Why this procedure? The answer is simple: out-of-plane deflections are dependent upon moments, and moments are dependent on out-of-plane deflections. In other words, a stable, nontrivial solution to a differential equation comes into the picture. Rather than forcing us to crack open the "Advanced Engineering Mathematics" textbook, ACI 318 allows us to use a numerical procedure (iterative calculations) until we find a solution that converges.

Most engineers will be inclined to implement this process using a spreadsheet, although when considering only one load combination, most solutions converge quickly and can be obtained by hand. However, once you start considering multiple load combinations and the ubiquitous trial-and-error design scenarios, automating the process becomes very attractive. Interestingly, a direct solution is available, not unlike the strength design process; but it becomes a challenge if we are on the threshold of cracking, since the equations are dramatically changed between the uncracked and cracked states. Hence, the procedure inherently requires that we consider whether the wall has cracked within each iteration, and we use different equations for peak out-of plane displacement accordingly.

Consider an example: An 8-inch thick concrete wall (f'_c = 4,000 psi) spans vertically 24 feet. One curtain of vertical bars is at the center of the wall with #5 @ 12 inches (assume effective depth d = 4 inches). Over a 1-foot wide design segment, unfactored loads include 1.6 kips of dead load and 0.5 kips of snow load, both at an eccentricity of 7 inches with respect to the wall centerline, plus a 30 psf outward wind load. Based on ACI 318-11 load combination 9-4 ($1.2D + 1.0W + 0.5S$), the wall is satisfactory, with M_u = 4.19 kip-ft/ft < ϕM_n = 6.21 kip-ft/ft. This calculation utilizes the effective area of steel drawn from the compressive axial load, the cracked moment of inertia, and other provisions of ACI 318-11 section 14.8. The calculated ultimate deflection of the wall at mid-height is approximately 4.6 inches.

Now consider serviceability and a correlating service load combination from ASCE 7-10 section 2.4: $D + 0.6W$, where W is taken as the same 30 psf wind load indicated previously. Following the procedure from ACI 318-11 section 14.8, the calculation for deflection appropriately incorporating the P-delta effect requires the iteration of deflections approach unless you are certain that the wall does not crack, in which case a direct solution can be found similar to the strength analysis. It turns out that the deflection is about 0.10 inches, a far cry from the 4.6 inches determined from the earlier strength calculation.

Why the major disparity? It stands to reason that the strength analysis and the serviceability analysis should yield different results, but by a factor of nearly 50? What are we missing? The answer is simple and was alluded to earlier: the serviceability analysis considers *whether* the wall has cracked, whereas the strength analysis *presumes* that it has cracked. For this example, the serviceability analysis shows a peak moment of 2.7 kip-ft/ft, while the cracking moment for this wall, based on the modulus of rupture, is 5.06 kip-ft/ft. Hence, the wall does not crack and essentially maintains its gross section properties and relatively high stiffness. Interestingly, the cracked moment of inertia for the strength analysis is only 34.7 in^4/ft, whereas the gross moment of inertia is 512 in^4/ft.

Considering this, coupled to the concept that the procedure for strength design presumes that the wall is cracked, it should not be surprising to see such a large difference in deflections between the strength design and serviceability design approaches. Closer corollaries in behavior between strength analysis and serviceability analysis can be observed in elements with relatively high loads such as jamb columns, which are more likely to crack under service loads.■

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