building **BLOCKS**

Test-Based Available Strengths for Aluminum Structures

Part 1

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This is Part 1 of a two-part series. This installment summarizes two methods of determining available strengths from testing of aluminum structures, presents equations, and includes selected plots of calculated safety factors. The next part will discuss and compare the results from the two methods.

Let's say you have some test data for an aluminum component or structure. Will it make a difference which method is used to establish an available strength with that data? In the Aluminum Association's 2020 *Aluminum Design Manual, Part 1 – Specification for Aluminum Structures* (Appendix 1: Testing [§1.3]), there are two methods for determining available strength from test data. This study included both the Allowable Strength Design (ASD) and Load and Resistance Factor Design (LRFD) procedures to compare the results from these two methods. This was an analytical study only – no tests were conducted. However, the author's prior use of these two methods, for a few individual sets of test data, indicated that allowable strengths from Method 1 tended to be more conservative than those from Method 2.

The number of samples considered ranged from 7 to 50 and the coefficient of variation from 4% to 20%. For Method 1, safety and resistance factors from the *Specification* are applied to calculated nominal strength values based on test statistics. Method 2 available strengths are found by applying calculated (using test statistics and other parameters) safety and resistance factors to test averages. Of the many possible input values for Method 2, the current study was restricted to default values for the parameters. Plots of Method 2's calculated safety factors provide a sense of the wide range of possible values. For the set of variables considered, this study (including Part 2) shows that available strengths based on Method 1 are generally, but not in all cases, less (by widely varying percentages) than corresponding values from Method 2.

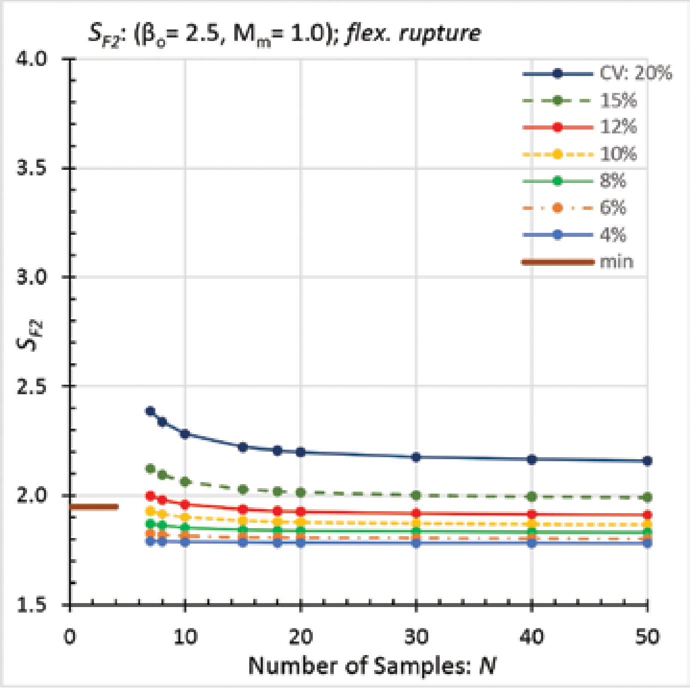

Figure 2. Safety factor S_{F2}: rupture of beams.

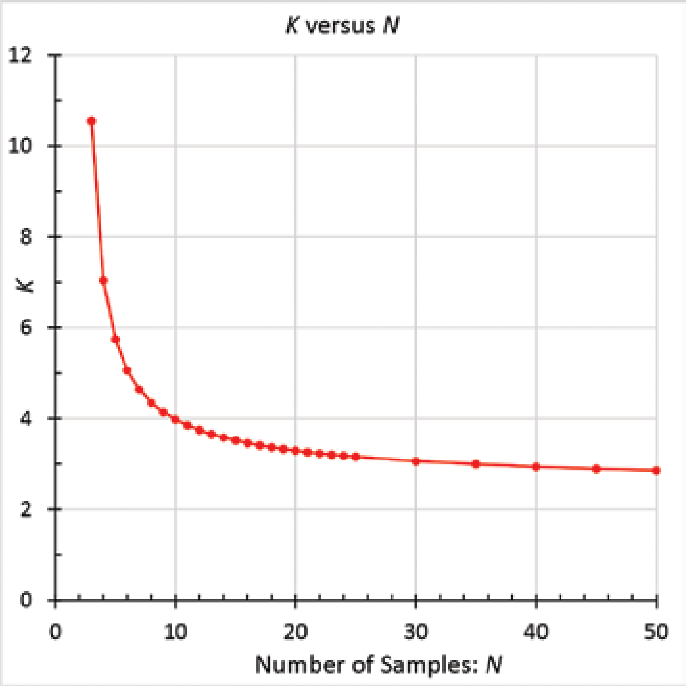

Figure 1. Statistical coefficient (K) vs. the number of samples (N).

Method 1

This method is simpler to use than Method 2 and has fewer input parameters. As given in the *Specification* for various limit states, the pertinent safety factors (Ω) for ASD range from 1.95 to 3.0, and the resistance factors (φ) for LRFD from 0.75 to 0.50. To find the allowable strength ($R_{1\Omega}$), this method uses a *calculated* nominal strength (R_{N1}), which per Method 1 is based on test statistics, divided by a safety factor chosen separately depending on the limit state. Similarly, the design strength ($R_{1\varphi}$) is the product of the resistance factor and R_{N1}. The nominal strength (R_{N1}) is a statistical lower bound (99% exceedance, with 95% confidence) on strength, which is based on test average (R_{TM}), samplte standard deviation (σ_x), and a statistical coefficient (K; *Figure 1*) that is based on the number of samples (N). Minimum N is 3. For $7 \le N \le 50$, K varies from 4.641 to 2.863. K accounts for uncertainty about the possible difference between the sample and population

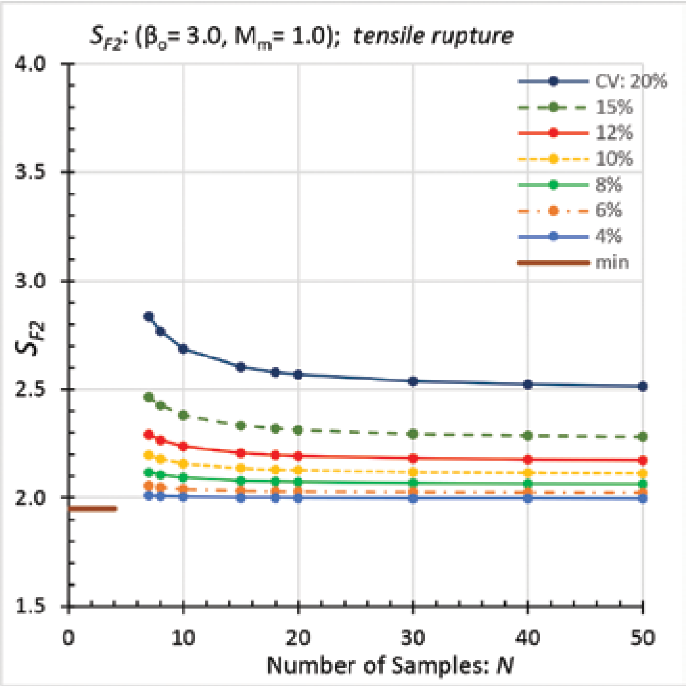

Figure 3. Safety factor S_{F2}: rupture of tension members.

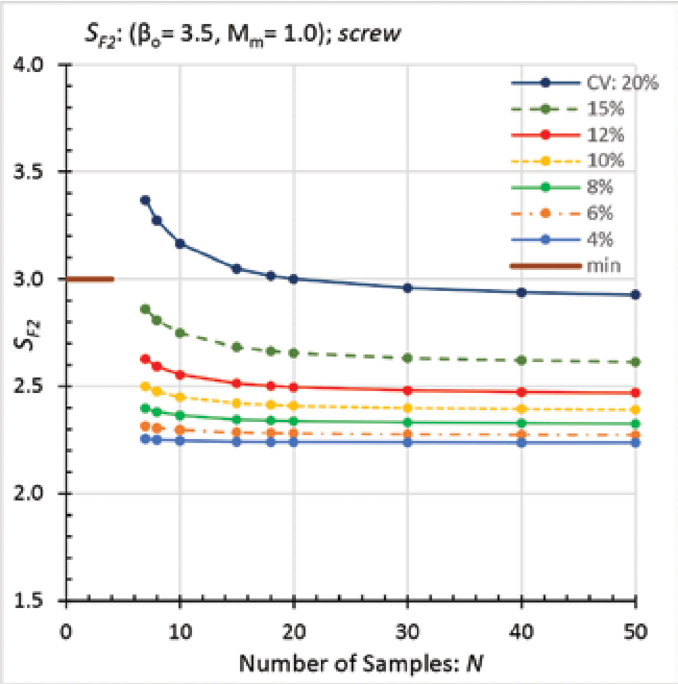

Figure 4. Safety factor S_{F2}: tapping-screw connections.

standard deviations; it increases at an increasing rate as N becomes smaller, especially for N less than about 20.

To compare Method 1 available strengths with Method 2 available strengths, for ASD and LRFD, the applicable equation for the non-dimensional ratio of available strength to average test strength was determined for each method. Variables include:

K = statistical coefficient
N = number of samples
R_{N1} = calculated nominal strength
R_{TM} = test mean (average) strength
σ_x = sample standard deviation
φ = resistance factor for LRFD
Ω = safety factor for ASD

The Method 1 allowable strength (ASD) is:

$R_{1\Omega} = R_{N1} / \Omega = (R_{TM} - K\sigma_x) / \Omega$ (Eqn. 1)

Now divide the allowable strength by the test average. Note that the coefficient of variation $C_V = \sigma_x / R_{TM}$:

$R_{1\Omega} / R_{TM} = (1 - KC_V) / \Omega$ (Eqn. 2)

The Method 1 design strength (LRFD) is:

$R_{1\Phi} = \varphi R_{N1} = \varphi (R_{TM} - K\sigma_x)$ (Eqn. 3)

Divide the design strength by the test average:

$R_{1\Phi} / R_{TM} = \varphi (1 - KC_V)$ (Eqn. 4)

Method 2

See the *Specification* for further Method 2 details. As an example, this method had previously been applied to data for screw pull-out from screw chases. For the current more general study, the default values of various parameters were employed:

- α (= 0.2): dead-to-live load ratio
- M_m (= 1.00 for rupture): material factor
- F_m (= 1.00): fabrication factor
- V_M (= 0.06): material variation
- V_F (= 0.05 for structural members and mechanically fastened connections; 0.15 for welded connections): fabrication variation
- V_Q (= 0.21): load variation
- β_o (= 2.5 for beams and columns, 3.0 for tension members and 3.5 for connections): target reliability index.

To determine allowable strengths (R_{2SF}) for Method 2, the *average* test strength is divided by a *calculated* safety factor (but not less than a minimum), which depends on many statistical variables. To distinguish the safety factor (Ω) in Method 1 from that in Method 2, the notation S_{F2} is used here for the Method 2 safety factor.

$S_{F2} = e^{\psi} (1.05\alpha + 1) / [M_m F_m (\alpha + 1)]$ (Eqn. 5)

where,

$\psi = \beta_o (V_M^2 + V_F^2 + C_N V_P^2 + V_Q^2)^{0.5}$

Here, $V_P = C_V$, which is the coefficient of variation for the test results, and $C_N = (N^2–1) / (N^2–3N)$. The minimum N is 4. For ASD, S_{F2}* is the greater of S_{F2} and the applicable value of Ω in the *Specification*. The allowable strength is:

$R_{2SF} = R_{TM} / S_{F2}^*$ (Eqn. 6)

For Method 2, the ratio of the allowable strength to the test average is:

$R_{2SF} / R_{TM} = 1 / S_{F2}^*$ (Eqn. 7)

The calculated resistance factor in Method 2 is denoted here as φ_2 to distinguish it from the resistance factor (φ) used in Method 1.

$\varphi_2 = 1.5\ M_m F_m / e^{\psi}$ (Eqn. 8)

For LRFD, φ_2* is the lesser of φ_2 and the applicable value of φ in the *Specification*.

The design strength is:

$R_{2\Phi} = \varphi_2^* R_{TM}$ (Eqn. 9)

For Method 2, the ratio of the design strength to the test average is:

$R_{2\Phi} / R_{TM} = \varphi_2^*$ (Eqn.10)

Safety Factors

Plots of calculated safety factors (S_{F2}) and the required minimums are shown in *Figures 2* through *5*. Each plot is based on a different

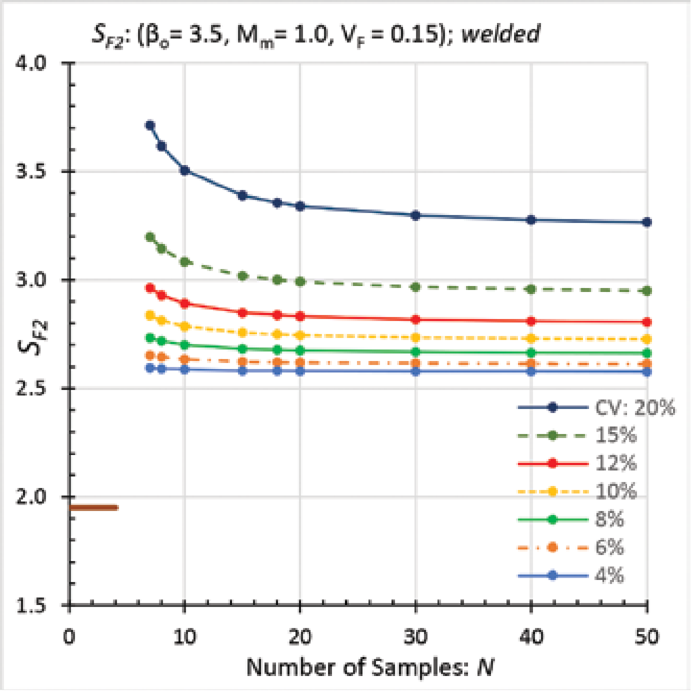

Figure 5. Safety factor S_{F2}: welded connections.

combination of β_o, M_m, and V_F. V_F equals 0.05, except for *Figure 5*, where it is 0.15. S_{F2} decreases as N increases (C_V constant) and as C_V decreases (N constant) for all plots. For large N and small C_V, the decrease in S_F is imperceptible in the figures. In these figures, a small number of samples (combined with intermediate or large C_V values), or a large C_V, typically results in a relatively large S_{F2}.

In each *Figure*, the range of calculated safety factors (S_{F2}) is:

- *Figure 2* (beam rupture): 2.39 to 1.78, but the required minimum is 1.95.
- *Figure 3* (rupture of tension members): 2.84 to 2.00, which exceeds the minimum of 1.95.
- *Figure 4* (tapping-screw connections): 3.37 to 2.24, but the minimum is 3.0, which governs over most of the calculated values.
- *Figure 5* (welded connections): 3.71 to 2.58, all of which exceed the minimum of 1.95.

Where a calculated safety factor (S_{F2}) exceeds the corresponding minimum (Ω), then S_{F2}^* equals S_{F2}. In this case, Method 2 determines an allowable strength that is less than R_{TM} / Ω.

Resistance Factors

Plots are not shown for calculated resistance factors (φ_2), but the *Table* provides a sampling of results for each condition.

If a calculated resistance factor (φ_2) is less than the corresponding upper limit (φ), then φ_2^* equals φ_2. In this situation, Method 2 provides a design strength that is less than φR_{TM}.

For both ASD and LRFD in Method 2, the available strengths are based on the *test averages*. However, the main body of the *Specification* bases available strengths on *nominal strengths* (R_N), which are in most cases less than test averages. This means that a test average divided by S_{F2}^* could produce an allowable strength that exceeds R_N / Ω. Similarly, a test average multiplied by φ_2^* could result in a design strength that is greater than φR_N.

Method 1 vs. Method 2

Note that $KC_V > 0$ for $C_V > 0$. Given this, *Equation 2* (Method 1's ratio of allowable strength to test average) is less than *Equation 7* (Method 2's ratio for allowable strength) if S_{F2}^* equals Ω. Similarly, *Equation 4* (Method 1's ratio of design strength to test average) is less than *Equation 10* (Method 2's ratio for design strength) if φ_2^* equals φ.

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References are included in the online PDF version of the article at **STRUCTUREmag.org**.

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Table of bounding values of φ_2 for LRFD.

Case	Calculated φ_2		Upper limit	Comment
	Min.	**Max.**		
Beam Rupture	0.63	0.85	*0.75*	Limit < max
Tension-Member Rupture	0.53	0.76	*0.75*	Limit < max
Tapping-Screw Connections	0.45	0.68	*0.50*	Limit < max
Welded Connection	0.41	0.59	*0.75*	Max < limit

References

Specification for Aluminum Structures (Part 1 of the *Aluminum Design Manual*), The Aluminum Association, 2020

Commentary on the Specification for Aluminum Structures (Part 2 of the *Aluminum Design Manual*), The Aluminum Association, 2020

Galambos, T.V. (1979), *Load and Resistance Factor Design for Aluminum Structures*, Research Report No. 54, Washington University, St. Louis, MO; (for the basis of Method 2)

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