Why It's Good to be a Lightweight

Geometrically Nonlinear Structures

By Peter Debney, BEng(Hons), CEng, MIStructE

Peter Debney, BEng(Hons), CEng, MIStructE, is a Chartered Structural Engineer and software specialist with over 20 years' experience specializing in computing applications, half of those in practice and the rest in engineering software, whether BIM, analysis or design. He is an application specialist for Oasys, concentrating on structural and crowd simulation software.

Apart from the weight, there is nothing lightweight about lightweight structures. With traditional structures, the loads are resisted by the stiffness in the beams, columns, and walls; with tension-only and compression-only structures, the overall form of the structure becomes critical. Get the form right and the structure can span huge distances with minimal material; get the form wrong and you are in trouble.

> *"Students … learned that nonlinear systems were usually unsolvable, which was true, and that they tended to be the exception – which was not true."*
> James Gleick (1988)

We can all remember, in structures 101 lectures, when we were told that a beam could have a pin at one end and a roller at the other, but it could not work if you added a pin in the middle as well. This is true if you want to keep the analysis linear, and not create any horizontal reactions, but is a hopeless approach if you want a bridge to cross 1.5 miles of river. The linear approach just will not work; it's time to take a nonlinear approach.

Many building structures are linear, or at least near enough to not worry about nonlinear effects. The beams and columns are kept within tight deflection limits and they tend to behave in a linear way. The truth is that all structures are nonlinear. It's just that the simpler linear analysis usually gives answers that are close enough for the majority of engineering design.

So what makes nonlinear different from linear analysis? One of the most important things to remember is that with linear analyses you establish equilibrium of the forces on the original geometry, but with a nonlinear analyses you get equilibrium of the forces on the deformed geometry. The problem is, you don't know what the deformation is until you have resolved the forces and you cannot resolve the forces until you know the deformed shape. All nonlinear analyses thus requires a certain amount of iteration.

Cables

While linear structures resist loads with bending stiffness, lightweight nonlinear tensile structures work by deflecting until the forces are in balance.

Take a look at a very simple example: a cable supporting a single point load at its centre. For extra simplicity, ignore the self-weight of the cable, strain hardening and strain limits. Concentrate purely on the point load and how the cable responds to it.

Before the load is applied, the cable is straight and unstressed. When the load is applied, the cable cannot resist: it has no bending strength, so it starts to deflect. As it deflects, it begins to stretch and tension is induced. As the cable is no longer perpendicular to the load, there is therefore a vector component of that tension that resists the load. As the deflection increases, so does the induced tension and angle until the deflection reaches the point where the load is exactly balanced by the parallel vector components in the cable tension (*Figure 1*).

This structure is nonlinear, as it has to deflect to carry the load. Don't forget that all structures have to deflect to carry load, but nonlinear ones such as this have to move a significant distance to reach static equilibrium.

Force/Action → Acceleration → Deflection + Resistance → Strain → Stress → Reaction → Equilibrium

The nonlinearity in the system is seen by plotting load against deflection (*Figure 2*).

Note that the final axial force in the cable is totally dependent on its final angle. Such a structure is also very sensitive to the support stiffness: as the supports move in, the deflection increases, as does the angle of the cable. This will result in a larger deflection but a lower stress in the cable. If the two parts of the cable become parallel (either by infinite deflection or the more useful option of moving the supports together until they touch), the axial force will be equal to exactly half the applied load. Note also that there is minimal lateral resistance to movement. You have made a pendulum.

Conversely, we can reduce the deflection by applying a pre-stress to the cable, so that the higher tension generates the required resistance at a shallower angle. In theory at least, you could achieve zero deflection with infinite pre-stress. Practical structures fall between these two extremes.

The picture starts to get more interesting when section and material properties are considered. Initially, one might surmise that the deflection of the cable is independent of the cable stiffness, but the reality is that tension will build faster in a material with a higher stiffness and thus reach equilibrium at a lower deflection. There is also the structure's behaviour when the stress in the cable reaches the material plastic yield limit: the cable might yield but the structure will still carry the load. Even though the cable cannot carry any more stress, the load is carried by a combination of the stress and the angle of the cables, so any further increase in load results in a much higher deflection. In theory, the cable can carry a load of twice the yield strength of the cable. However, that would require the cable to deflect to infinity; the strain limit will have been reached long before that.

So with one point load on a cable you get a V shaped result. Adding more point loads or making the load uniform will pull the cable into the classic catenary shape, familiar to us from suspension bridges.

Figure 1. Diagram of deflection and load.

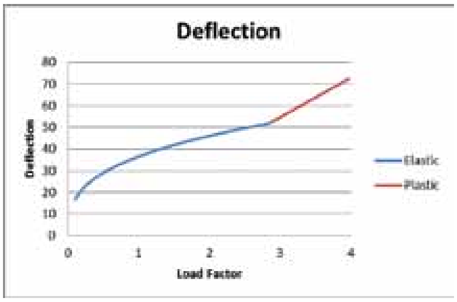

Figure 2. Load-deflection.

Figure 3. Spiders web.

Figure 4. Climbing frame.

Cable Nets

A single loaded cable, such as a catenary, is stabilized out-of-plane by gravity and possibly additional factors such as the bridge deck. However, such structures are still vulnerable to sway, whether induced by wind or pedestrians. A solution to this problem (though not normally for bridges) is to have cables going in multiple directions, so sway in one direction is resisted by cables at other angles, providing what is called a cable net. These are actually common structures in nature. It is likely that there are tens if not hundreds in your house and garden, spun by spiders (*Figure 3*).

If such a cable net is horizontal and loaded, it will deflect down with an essentially catenary shape, giving resistance to gravity loads. There is still a problem: what about uplift forces on such a cable net if clad? Suction would be resisted purely by the self-weight of the structure, which is minimal. As catenaries are good at resisting load in the direction of the curve, the solution is to have the cables in one direction curve down to resist gravity loads, and those in the other direction curve upward to resist suction loads. This double-curved hyperbolic surface is characteristic of many cable nets and all engineered fabric structures, as the shape naturally gives stiffness in all directions. The alternative is to provide cables in all directions, which might not give a practical roof but makes for great climbing frames (*Figure 4*).

Velodrome

So what does such a double curved surface look like? An excellent example is Expedition's award winning Velodrome for the London 2012 Olympics, famous for its "Pringle" shaped roof. Beneath the 16,146 square yard (13,500 m2) cladding lies a cable net, efficient enough to keep the total weight down to a mere 12 pounds per square foot (60 kg/m2). Despite its lightness, the roof is still stiff enough to keep the lighting from shaking, something that the cyclists would find extremely distracting. The shape also has the advantage of minimizing the volume of the space and thus reduce heating and cooling loads while at the same time maintaining audience sight-lines.

On the other hand, spiders instinctively make their webs in a single plane, giving them a flexibility undesirable in most engineered structures. From the spider's point of view this is exactly what they need, as they are, like safety nets, designed to catch and absorb the energy from fast moving objects. A double curved spiders web may be too stiff and thus allow flies to bounce off before the glue has a chance to work.

As tension structures are very sensitive to movement at the supports, the Velodrome roof needed a stiff steel compression ring, which was in turn borne by raking trusses that also supported the seating. The trusses in turn were rigidly mounted on the concrete

Subject Sources and Further Reading

There are many sources available online and in the bookshop on the subjects covered in this article. Here are a few:

Gleick, J. (1988). *Chaos: making a new science*.

Motro, R. (2003). *Tensegrity, Structural systems for the future*. London.

Oasys Software. *GSA Suite*. [online] Available at **www.oasys-software.com/GSA**.

The Structural Engineer, 2012. *Olympic Structures for London 2012*. The Institute of Structural Engineers. Available at **www.istructe.org/journal/volumes/volume-90/issues/issue-6**.

Expedition Engineering. *The London Velodrome*. [online] Available at **www.expeditionworkshed.org/workshed/the-londowww.n-velodrome**.

Buro Happold Engineering. *London 2012 Olympic Stadium*. [online] Available at **www.burohappold.com/projects/project/london-2012-olympic-stadium-132**.

Mott MacDonald. *London 2012 shooting events venue*. [online] Available at **www.mottmac.com/events/london-2012-shooting-events-venue**.

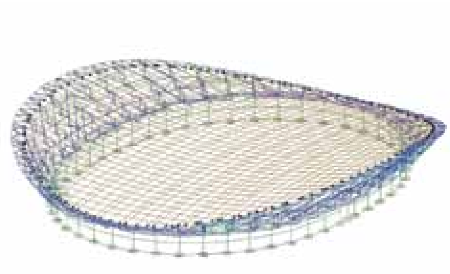

Figure 5. GSA model of Velodrome. Courtesy of Expedition Engineering Ltd.

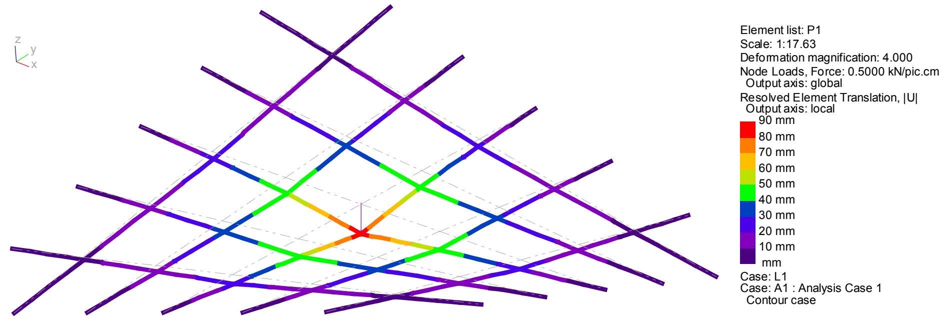

Figure 6. Hyperbolic paraboloid double-ruled mesh.

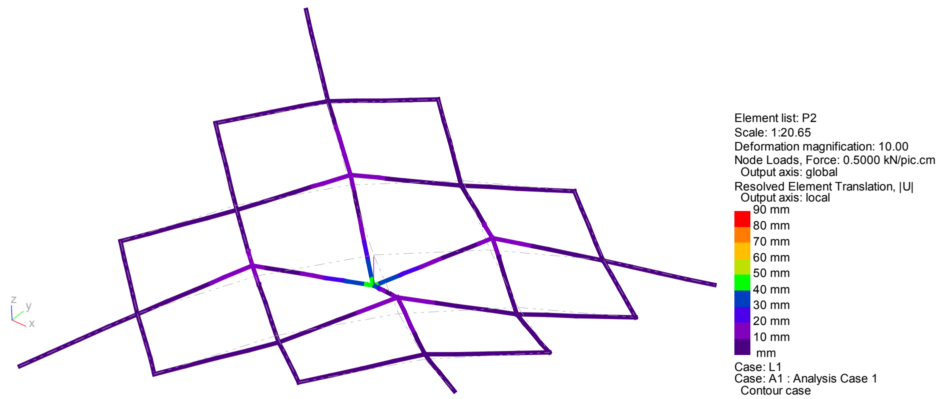

Figure 7. Hyperbolic paraboloid mesh.

base structure. Designers used GSA Analysis modeling software (Oasys) throughout the design process, from form finding the cable net to static analysis to checking the vibration characteristics of the completed building (*Figure 5*).

Forming a Hyperbolic Surface

Unlike most double curved surfaces, hyperbolic surfaces have a curious property: you can make them entirely out of straight lines. We are all familiar with single-ruled surfaces such as cylinders, where you can define the surface with a series of straight lines all going in the same direction. Hyperbolic surfaces are double-ruled surfaces, meaning that they are formed from two series of parallel lines. The classic version of this is the hyperbolic paraboloid, or hypar for short, which you can form by twisting a rectangular plane (*Figure 6*).

Although the Velodrome roof is a hyperboloid, it is slightly different as the cables were at 45° to the ruled surface (*Figure 7*).

Both options have the same surface and about the same quantity of material, but the latter is twice as stiff due to the curved profile of the cables, halving the deflection. This means that the longer span cables actually deflect less.

Rigid hyperboloid structures were first used by the Russian engineer Vlaadimir Shukhov in the 1890's with his lattice tower in Polibino, Lipetsk Oblast. He is most famous for his Shabolovka Radio Tower. The form has been subsequently used for architectural towers such as Zhou Ruogu and the Kobe Port Tower (*Figure 8*), but also to give the humble cooling tower its buckling resistance.

Figure 8. Shabolovka Tower, Zhou Ruogu, Kobe Port Tower.

Hyperboloids are not the only form of double-curved surface used with lightweight structures. The basic forms also include the Conic and the Barrel Vault (*Figure 9*). There is also the dome, which will be discussed in Part 2 of this article.

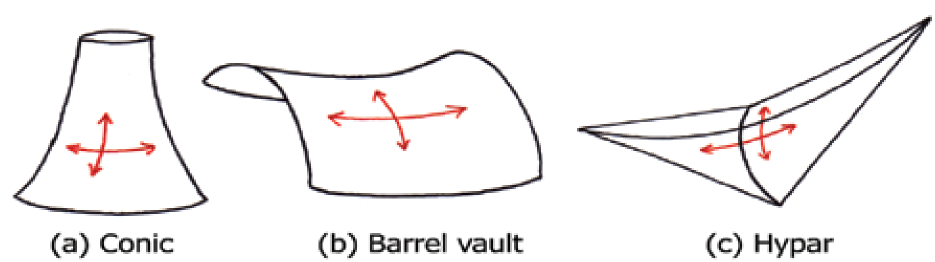

Figure 9. Hypar, Conic, and Barrel forms.

Fabrics

The boundary between fabrics and planar cable nets can be a blurred one. Structurally, one of the most important differences is that fabrics are woven while cable nets tend to be layered. This gives rise to the fabric's warp-weft interaction, which means that when you tension one direction more than the other, the fibres in that direction straighten and increase the kink in the other. This gives fabrics both an unusual stress-strain relationship and Poisson's ratio. Fabrics are thus sensitive to the balance of pre-stresses in the two principle directions; the fabric will wrinkle as a whole if the pre-stress is much higher in one direction than the other (*Figure 10*).

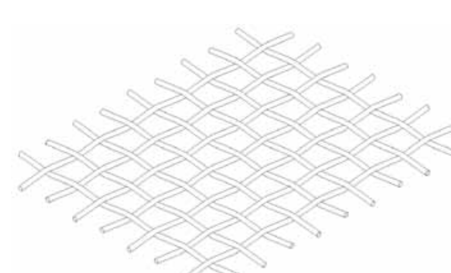

Figure 10. Fabric weave.

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Figure 11. 2008 Olympic Aquatic Centre, Beijing. Courtesy of Arup.

Because of this arrangement, fabrics are principally stiff in the warp (principle fibre) and weft/fill (secondary fibre) directions, and have minimal stiffness diagonally. Coating the fabrics can increase this stiffness, but the low shear stiffness does aid the fabric's ability to twist into the required double curved surfaces.

Foils

A step further on from fabrics are foils, which are isotropic plastic sheets made from materials such as ETFE (ethylene tetrafluoroethylene) and in use on iconic structures such as The Eden Project and Beijing 2008 Water Cube (*Figure 11*). Foils are generally used in inflated pillows, so each cladding panel is actually two or three separate layers supported by pressurized air. Wind loads on one surface are carried through the contained air to load the opposite face, so the whole remains in tension. The air can be heated to prevent snow loads. Unlike fabrics, foils have a good shear strength; they have to yield under load to achieve their final form, though form finding and determining the correct cutting patterns goes a long way to minimize this.

Edge Conditions

Because the pre-stress is crucial to ensure the tension and hence stability of fabrics, it is important to consider how to achieve this pre-stress. Fabrics require an edge support, which can either be solid, such as a beam, or flexible, such as a cable. With flexible edges, the cable's curvature (or "set") is dependent on the balance in the pre-stress between the cable and fabric. Apart from structural considerations, this set has quite an impact on the aesthetics of the fabric structure.

The balance of the pre-stress in the fabric is also crucial and must be in harmony with the fabricated surface. A correctly tensioned fabric will be smooth, as can be seen in Arup's Marsyas sculpture at the Tate Modern in London. Unbalanced tensions will cause wrinkles in the surface, known as Heugen's Tension Fields (*Figure 12*).

Figure 12. Masyas. Courtesy of Arup.

Munich 1972

The modern science and engineering of fabric structures was pioneered by Frei Otto, with his roof to the Munich Olympic Games being a major landmark in the industry. In rejection of the heavy wartime architecture of Nazi Germany, Otto aspired to make modern architecture as light as possible, in both senses of the word. The Munich roof achieved this by using both a minimum of material and maximum glazing (*Figure 13*).

Frei Otto's seminal work has continued to inspire and influence Olympic architecture. While the end effect is quite different, Mott MacDonald's 2012 Shooting Gallery façades in London are based on exactly the same principles.

Otto and his team needed to determine the geometry using physical models, with careful measurements feeding into hand calculations. Mott MacDonald, on the other hand, was able to use Oasys GSA for both the form finding and subsequent static analysis.

Tensegrity

So far we have looked just at tension structures, but now let's start to mix in compression with the rather interesting group of structures known as Tensegrity.

Buckminster Fuller coined the term as a portmanteau of Tension and Integrity, and described them as an "island of compression in an ocean of tension" (**http://bfi.org**). Rene Motro went a little further when he said that,

Figure 13. 1972 Olympic Main Stadium, Munich.

"A tensegrity is a system in stable self-equilibrated state comprising a discontinuous set of compressed components inside a continuum of tensioned components." And artist Kenneth Snelson (**http://kennethsnelson.net**) went further still with "Tensegrity describes a closed structural system composed of a set of three or more elongate compression struts within a network of tension tendons, the combined parts mutually supportive in such a way that the struts do not touch one another, but press outwardly against nodal points in the tension network to form a firm, triangulated, prestressed, tension and compression unit." Now we see tensegrity at work in superb structures like Brisbane's Kurilpa Bridge (*Figure 14*).

It's a cantilever beam, but where is the compression flange? Each element is either in pure tension or pure compression (ignoring the small bending on the struts from the self-weight), so the load path is not conventional.

Like cable nets, pre-stress is essential to the stability of these structures to give them sufficient stiffness. You can make the simplest

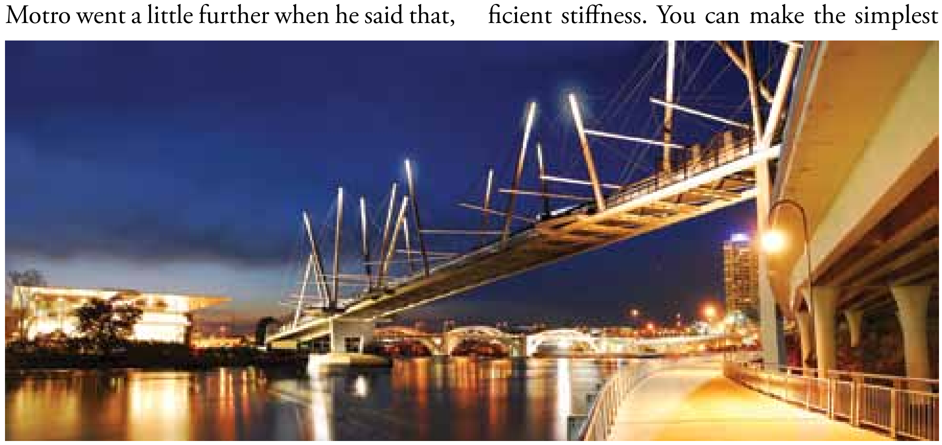

Figure 14. Award-winning tensegrity in Brisbane's Kurilpa Bridge. Courtesy of Kenneth Snelson.

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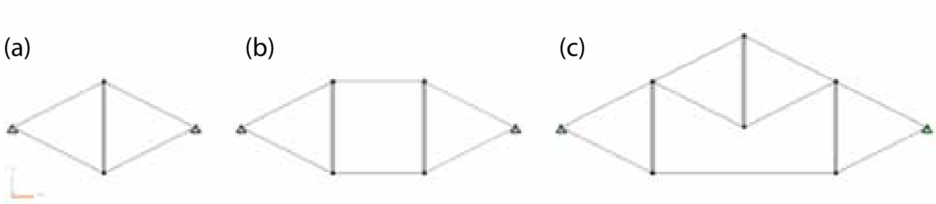

Figure 15. Tensegrity units.

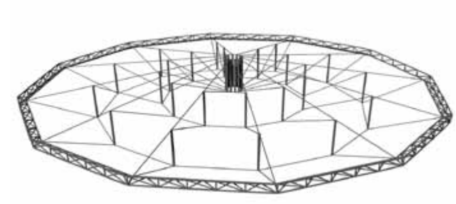

Figure 16. GSA tensegrity roof. Courtesy of Arjan Habraken, TU Eindhoven.

tensegrity work with just one strut and four cables (*Figure 15a*) or slightly more complex with two struts (*Figure 15b*). You can then start to combine these basic units to create more complex forms (*Figure 15c*). While these are stable in plane, they are unstable out of plane, so you will need additional cables or a full wheel (*Figure 16*).

With a suitable compression ring, tensegrity structures have the ability to cover huge spaces such as stadia with minimal weight, usually by forming inscribed hoops, each one supporting the next. There are two classic forms to tensegrity roofs, known as the Geiger, where the cables are arrayed radially, and the Fuller, which is triangulated for improved stiffness.

2012 Olympic Stadium

An excellent and rather subtle version of the tensegrity roof is Buro Happold's 2012 Olympic main stadium. Here the fabric infill panels disguise the boldness of this structure, with a single main cable loop tensioned and supported by radial cables from the steel frame. This cable, in turn, supports the 14 lighting rigs, each weighing 50 metric tons. These rigs are stabilized by back stays and a tension ring of their own (*Figure 17*).

The programme did not allow for complete scaffolding of the compression ring, which was not stable until it was complete, so each stage in the erection sequence required separate GSA analyses to ensure stability and also to determine the locked-in stresses to take into the next stage. Further analyses were also required for each stage in tensioning the cable hoop, and then erection of each lighting rig and the fabric roofing (*Figure 18*).

Conclusion

Though their lower stiffness or deflection requirements do not make them suitable for every application, lightweight nonlinear structures already enable us to span huge distances with minimum materials. Part 2 of this article will look into the world of compression-only structures.

Acknowledgements

My thanks go to Matthew Birchall and his team at Buro Happold for the information on their 2012 Olympic Main Stadium. My thanks also go to Andrew Weir at Expedition for the information on their 2012 Olympic Velodrome, the various engineers at Arup that I have worked with over the years, and especially to my colleagues at Oasys for both creating the GSA software and for answering my many questions.▪

Figure 17. London 2012 main stadium. Courtesy of ODA.

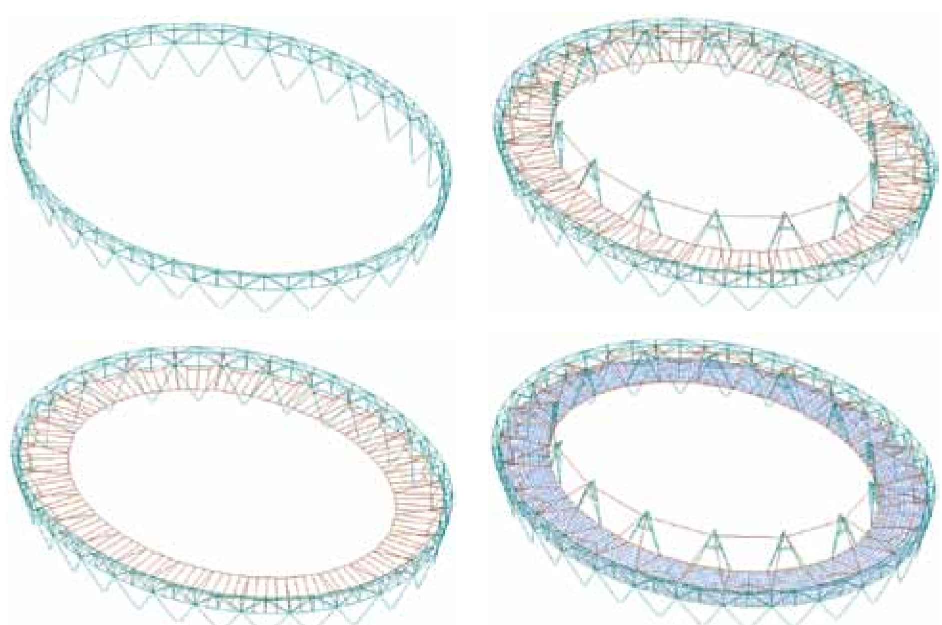

Figure 18. Main stadium GSA model. Courtesy of Buro Happold.