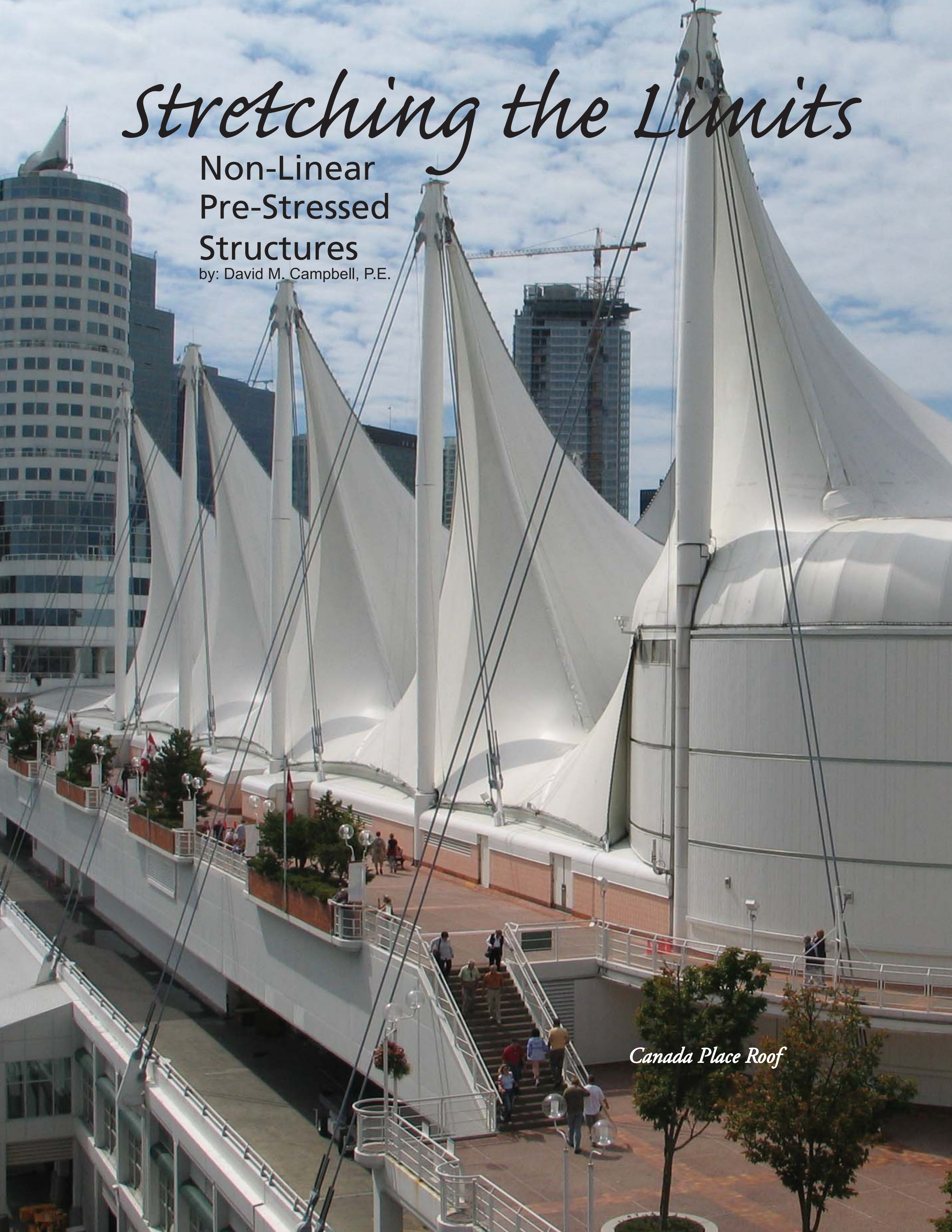
Stretching the Limits

Non-Linear Pre-Stressed Structures

by: David M. Campbell, P.E.



Canada Place Roof

Experience with tensile membrane structures provides insight into non-linear structural behavior. Design of structural systems intentionally eliciting non-linear large deflection behavior has achieved benefits. This has lead to the design of unique tension based, under-determinate and/or prestressed spatial systems.

The principles of designing with geometric stiffness are discussed below. This includes prestress as a property of a structural system, and deformation as a means of load response attenuation. These attributes of tensile membrane structures can be developed and exploited in a wide variety of tension based systems.

Non-Linear Response

Structures respond to applied load by changes in internal stresses and by displacement of internal forces.This latter response, "geometric stiffness", is ignored in linear structural theory with the presumption of "small" deflection. As this term is a function of both deflection and the internal force, it is inconsequential when this product is small in relation to changes in stress. The introduction of prestress in an otherwise conventional structural system can increase the internal forces sufficiently to make this term significant, but this simplification serves well for most structural systems where it is second order.

For "large" deflection structures, such as tensile membranes, the displacement of internal forces is always significant. There are structural systems with many non-extensional modes of deformation which are resisted solely by geometric stiffness. In some tensile membrane structures, the geometric stiffness is greater than the elastic or extensional stiffness. These structures cannot be analyzed without due consideration of this nonlinear behavior.

Experience with these systems leads to a general perspective in structural design, where nonlinear behavior is considered and indeed encouraged. Considerable advantage can be realized in tension based systems. Member/element prestress can be considered a design parameter as important as the elastic member/element properties.

Tensile Membrane Structures

Tensile membrane structures primarily fabricated from coated textiles, such as PTFE coated fiberglass or PVC coated polyester, rely on geometric stiffness. Textiles are generally good in tension and have limited shear stiffness. These material characteristics can be exploited by allowing movement and flexibility of the structure. Most structural applications have been for roofs and enclosures, as this use is most tolerant of large deformations.

There has been a steady trend to reduce the design prestress of large tent-form tensile structures. For structures that rely upon geometric stiffness for much of their load resistance, this results in increased deformations. The design rule of thumb 25 years ago was to set prestress for a permanent fabric membrane at the maximum permissible level, nominally 1/10th of the tensile strength of the material. The intent was to maximize the initial geometric stiffness aimed at reducing deformations, and to prevent flutter and other dynamic instabilities. As experience was gained with large flexible systems, larger deformations were permitted and initial prestress values were reduced.

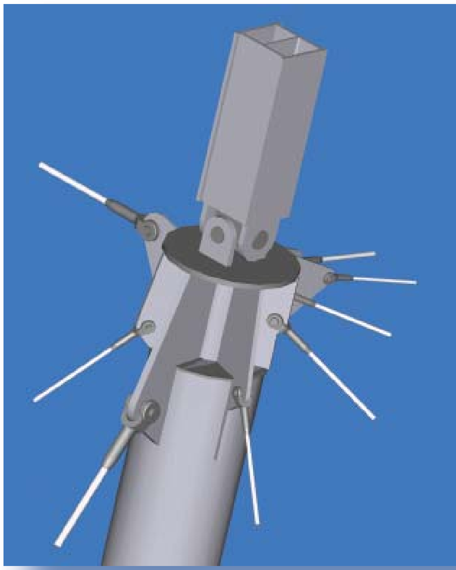
Construction advantages of lower prestress include:

• Benefits are achieved in the installation and stressing of the structure.

• As the initial geometric stiffness is reduced, greater deformation is required for the structure to resist a given load condition. In tent-form tensile structures which consist of a pre-stressed continuum, this generally means a larger portion of the structure is engaged in resisting a specific load distribution. A consequence is that a greater portion of the system is engaged in resisting local loads. The result is that the effects of load concentrations are attenuated in the system. Said in another way: local loads are resisted by local deformations rather than local changes in stress.

Table 1 - Design Prestress for Large Tent-Form Fabric Structures

Project	Clear Span	Prestress	Year
Haj Terminal, Jeddah Saudi Arabia	65 meters	123 kN/cm	1980
CanadaHarbour Place Vancouver, BC Canada	71 meters	61 kN/cm	1986
Ja-Yi Gymnasium Roof Ja-Yi County Taiwan	50 meters	35 kN/cm	1996



Detail 1– Articulated Mast Detail, Nautica Canopy, Cleveland, OH

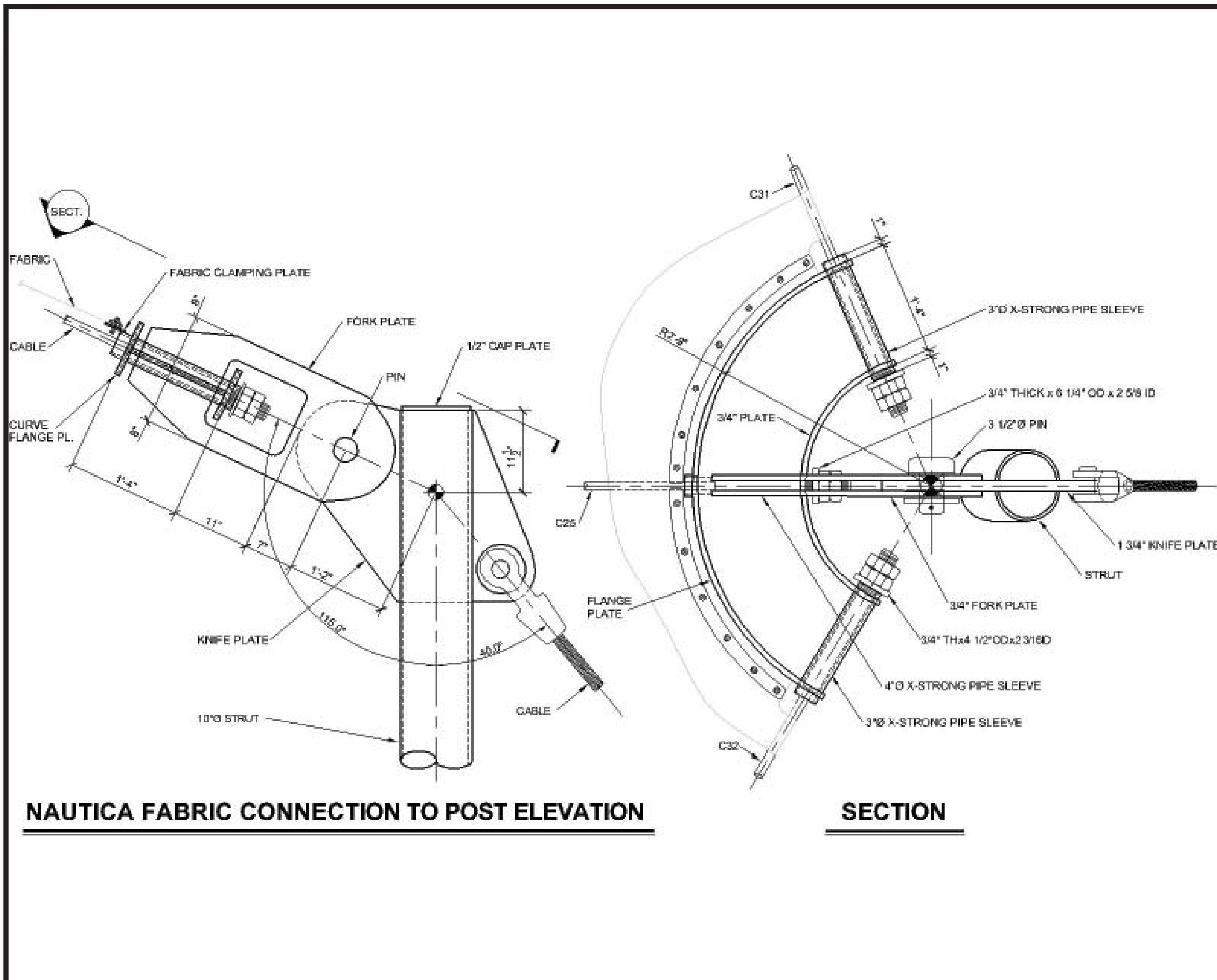
• With the major caveat that the deformations remain within serviceable bounds, such structures can be remarkably insensitive to load distribution. Deformations may in fact be quite large in contrast to either compression or flexure based structures. For example the maximum deflection of the Canada Harbour Place roof under non-uniform drifted snow is predicted to be nominally 2-meters on a 71-meter span. The Ja-Yi Gymnasium roof has maximum deflection of 1.62-meters from the design wind uplift on a 50-meter span.

Design Issues for Roof Structures

1. Design of these structures must carefully consider the movements and defor-mations. Members must be able to move to accommodate system deformations. Often multi-axis pins are required. *See Detail 1.* Connection of fabric membrane and rigid members requires consideration of relative movements of the connected components. *See Detail 2.*

2. Most structural membrane materials in use are coated textiles. Textiles have good tensile strength, but most are poor in tear. Care in detailing of fabric connections to cables, and framing members to allow for sufficient movement without pinching or otherwise initiating tears.

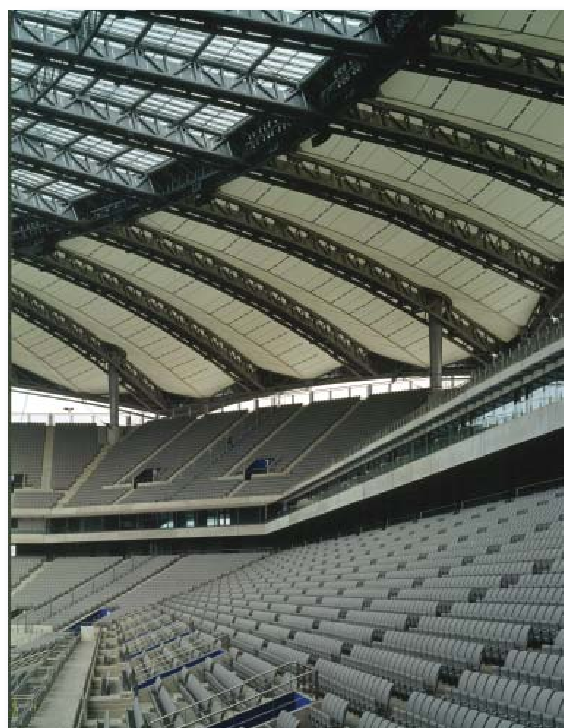
3. Allowing or encouraging large deformation is not without pitfalls. While membrane stresses are less dependent on specific load distribution, deformations are load distribution specific. Assuring serviceability in roof structures includes concerns with drainage and ponding of



Detail 2 – Cable and strut connection, Nantica Canopy

deformed roof forms, as well as dynamic behavior. Ponding instability caused by rain can be addressed analytically. In practice this is rarely necessary, for if a portion of a tensile membrane roof surface is found not to drain under relatively small load, it is probable that it will pond. Generally, the stiffness of the membrane structure will be found to increase with deformation, but usually at an inadequate rate to prevent overload when an adequate supply of water to the region in question is available. In such instances, the structure's shape (form, prestress and support conditions) will typically require modification.

4. Prediction of snow induced ponding is much more difficult, and generally can only be addressed qualitatively by review and evaluation of both the deformed roof surfaces and the potential accumulation for snow and water loads in any susceptible areas. The difficulty of design with respect to this issue begins with prediction of real snow load distributions for tensile membrane structures. Model codes and standards such as ASCE 7 provide good definition of drift loads and sliding snow surcharges for a variety of roof geometries. However, as with definition of pressure coefficients for roof wind loads, these do not apply to the unique shapes and forms of most tensioned membrane structures. Many of these tensile forms create unique drifting patterns, and because of the surface characteristics of most structural membrane materials, potential consolidation of snow loads from sliding. Prediction of specific snow load distributions are further complicated by the deformation of the structure under load, which can be sufficient to effect the concentrations of snow and ice load due to sliding as well as melt water saturation. Generally these effects cannot be readily modeled or quantitatively studied in the context of project design. Careful qualitative evaluation of tensile membrane structure behavior and deformations is required where snow events are



Seoul 2002 World Cup Stadium, Seoul

Ja-Yi Gymnasium Roof: 30% of prestress of 1980

likely. As with conventional roofs, sliding can be mitigated by employing snow cleats as was done on the fabric roof at Seoul 2002 World Cup Stadium.

"Pleated" Membranes

Adequate local curvature in tensile membranes is essential. In the tent-form structures, this generally means finding an anticlastic prestressed form with appropriately small radii of curvature. This is not always possible and other means of assuring suitable strength are necessary. Of course, it is not the prestress curvature that is important but rather the curvature of the membrane in its deformed condition under load which dictates the membrane stress.

"Pleated" membrane structures present an interesting case in point. The first structure to employ this system is the

"The design solution was to "pleat" the membrane..."

roof of the Lindsay Park Sports Centre in Calgary, Alberta, completed in 1983. The Myao Li Arena roof , 2000, employs the same arrangement. The roof membrane was employed in a manner that was a departure from previous tensile structures. The radii of the saddle surfaces which comprise the roof were too large for the strength of PTFE coated fiberglass. The design solution was to "pleat" the membrane by pulling it up out of the surface of the cablenet saddle with ridge cables. While this ensured regular reinforcing of the membrane with cables in the sagging direction, the radii of curvature of the membrane was still very large. However, each "pleat" has sufficient surface length from ridge to ridge, or valley to valley, which combined with the relatively soft boundary support of cables allows the membrane to develop significant local curvature under load. In this manner, the membrane always spans in the same direction by reversing curvature with reversals in load. The softness of the cable boundaries of the "pleat" panel assures that the membrane never goes slack in this cycle. The prestress form's radii of curvature are of little use in predicting membrane stress under load.

The use of "pleated" tensile membrane was central to the realization of the first tensegrity type domes, the Geiger Cabledomes, with 6 of 8 of these structures clad with fabric in this manner. The roof canopy structure of the Seoul 2002 World Cup Stadium employs PTFE coated fiberglass fabric in this form. In this case, the primary structure of the canopy is a spatial steel network that supports the ridges of the fabric membrane, As this structure forms a quite stiff boundary for the fabric panels, it was necessary to use a relatively elastic fiber-core wire rope valley cable to allow the panels to find suitable curvature under load.

Tensegrity

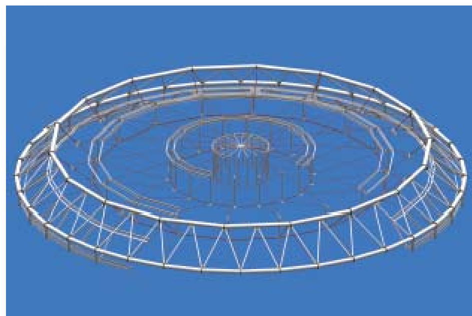
The principles of tension membrane structures are applicable to other structural systems, such as "tensegrity" type domes. The Geiger Cabledome, the first tensegrity type dome to be realized, is "under determinant" from the perspective of small deflection linear structural theory. As such, the system is reliant upon "large deflection" geometric stiffness in resisting non-uniform loads to be serviceable.

Seoul 2002 World Cup Stadium, Seoul: Pleated membrane form.

Seoul 2002 World Cup Stadium: Canopy cable suspended from 16 masts

The geometric stiffness is relatively large in non-extensional modes of deformation, as the tension in the hoops is large. The tension hoops resist out-of-plane deformation. Interestingly, by interconnecting all radials in the system, changes in hoop tension results a response of the entire cable and strut network. While such nonlinear behavior is referred to as "large deflection" behavior, deflections need not be much greater than those typical of conventional structures. Cabledome structures well illustrate this point. In the case of the Crown Center Coliseum roof, the maximum deflections under unbalanced load conditions are 200-millimeters to 250-millimeters on a span of 99.7-meters.

"The tension hoops resist out-of-plane deformantion."



Crown Center Cabledome

Many of the Cabledomes have relatively little prestress, i.e. the member forces are small when the system is not subjected to load. The majority of the hoop tension in the self-weight condition is a consequence of the self weight. The attendant geometric stiffness applies to changes in load from the self-weight condition. The conventionally clad Crown Coliseum is stiffer than other membrane clad Cabledomes of similar size such as the roof on Taoyuan Arena, completed 1994, or the Olympic Fencing Arena Roof in Seoul, because of the greater mass of its rigid roof panels.

Redbird Arena, ISU, Normal, IL: 1988-First Cabledome in USA clad with pleated fabric membrane

Non-Linear Thinking

Intentionally developing structures to exploit nonlinear geometric stiffness raises some interesting questions which require further thought:

1. Can Load Resistance Factored Design methodology be applied to structures which exhibit non-linear behavior? Super position of structural response is meaningless. Factoring prestress in a system changes an attribute of the system, and affects its response to applied load. Understanding the structural condition caused by factored loads may not necessarily provide insight into service conditions.

2. Deformations must be understood throughout the range of service conditions and addressed in design.

3. Prestress must be reliably developed and verified, as it is a critical aspect of the structural design. Loss of prestress and the ability to re-stress can be an issue.

4. Dead load deflection has little significance. The deformed shape under prestress and dead load is the typical condition of the structure, and is usually the initial condition for service load states.

5. Fabric membrane materials are generally selected by tensile strength, but most failures are initiated by tearing. Detailing to minimize stress concentrations and tearing is critical.

6. The dynamic behavior of systems with large geometric stiffness is difficult to address as the system stiffness varies with amplitude of deformation.▪

Crown Center: Structural System Rendered

Crown Center Construction

David M. Campbell P.E. is a structural engineer and managing principal of Geiger Engineers. The firms' engineering practice is dedicated primarily to special structures, long-span structures, large mechanisms, and sports facilities. Mr. Campbell has published numerous technical papers on long-span and tensile membrane structures and has written extensively about sports facility structures. In his 25 years of practice he has been involved in the design and construction of a wide variety of non-linear structures world wide.

Terminology

- PTFE: polytetrafluoroethylene. A polymer better known by Dupont's tradename Teflon.
- PVC: Polyvinylchloride
- Anticlastic: Double curvature where the principal curvatures have opposite signs as in a saddle-shaped surface.
- Tensegrity: Term coined by Buck-minster Fuller to describe "tension integrity" structures comprised of continuous tension elements and discrete discontinuous compression members.

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