ENGINEER'S NOTEBOOK

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Concrete Column Design

Back to the Basics

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Often a summary refresher helps keep us grounded in the fundamentals of elements that we commonly design. Owing to many requests from peers, this article is provided as a summary of the steps that may be taken for the development of a typical reinforced concrete column interaction diagram.

The methodology outlined below reflects the provisions of ACI 318, but it is not the only viable method. In fact, ACI 318 does not explicitly require an interaction diagram for column design. However, most structural engineers understand that such a tool is the most convenient form of expressing the nominal axial and flexural capacities, as well as the best tool for helping us know how axial loads and bending loads influence and affect one another.

Assume, just for the sake of argument, that the factored load effect for the design of a tied concrete column is a trivial matter, and that we have the results for M_u and P_u. The next step that we might follow would be to examine a generated interaction diagram and see whether our interactive load (represented by M_u and P_u) falls within the capacity boundary of our trial column.

Let us further assume that we do not have the benefit of an interaction diagram and thus are 'starting from scratch'. You may recall that an interaction diagram for a reinforced concrete column may be developed by examining a series of strain conditions at one surface of the column. These strain conditions are arbitrary, and are selected on the basis of providing the most descriptive capacity boundary that can conveniently be determined. At the opposing surface, an ultimate concrete compressive strain of 0.003 is assigned. This is meant to represent the strain in the concrete at ultimate compressive failure and is assigned this value regardless of concrete strength. *Figure 1* illustrates the establishment of the fixed 0.003 strain value at the compression edge while the layer of reinforcement at the opposite surface is subject to a series of strain conditions.

The series of strains (tensile and compressive) are then assigned to the layer of reinforcement opposite the concrete compressive failure surface. For any one particular level of strain that we have arbitrarily assigned, we can follow ACI 318 criteria and connect the two opposing points of strain on a diagram with a straight line, thus assuming that the strain is distributed linearly across with column width. This makes for simple calculation of the strain in the remaining layers of reinforcement, using the simple formulas for similar triangles that we learned in high school math.
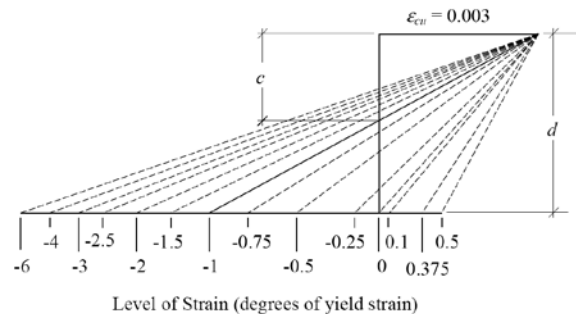

Figure 1: Superimposed series of strain diagrams.

The result is a strain for each and every bar in the column as it correlates to the level of strain that was arbitrarily assigned to the layer of reinforcement opposite the compression surface. This also helps us know where the theoretical neutral axis for the column is for this strain condition, which occurs where the aforementioned line intersects the vertical axis of the strain diagram. Once the strain levels and the neutral axis are known, the design may proceed to the next step.

By Hooke's Law, stress is equal to strain multiplied by the material's elastic modulus. Multiplying the strain for each layer of reinforcement by 29,000 ksi yields the corresponding stress. This must be truncated to the yield stress for any results lying outside of the elastic range of behavior for the reinforcement (typically 60 ksi). For the concrete acting in compression, the depth of the compression zone is related to the neutral axis by the β_1 value, which is a function of f'_c and ranges from 0.65 to 0.85 (0.85 for f'_c of 4000 psi or less). The stress in the concrete has a default value of $0.85f'_c$ and is assumed to be distributed uniformly over the entire compression region. *Figure 2* illustrates the relationship between the strain and stress diagrams for one iteration of interaction diagram development.

Once the stress in each layer of reinforcement is known, as well as the dimensions of the concrete compressive stress region, the resultant forces in each are calculated simply by multiplying the stresses by the respective areas. Summing the result yields a total force P_n, the nominal axial capacity of the column as it
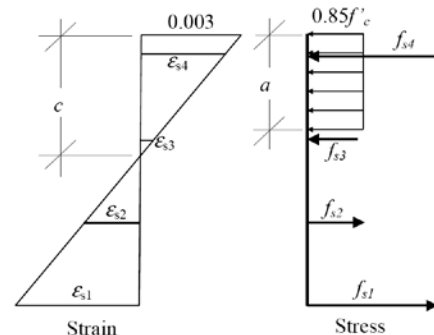

Figure 2: Assumed strain and stress diagrams for reinforced concrete.

correlates to this level of strain. Multiplying these same forces by their relative distances from the centroid of the gross column section and summing the result yields the nominal moment capacity M_n.

The final step for this one iteration of design is to determine the strength reduction factor that is appropriate for the level of strain under consideration. This is a function of the net tensile strain arbitrarily assigned earlier; it has a value of 0.9 for net tensile strains of 0.005 or more and a value of 0.65 for net tensile strains of 0.002 or less. Intermediate values are linearly interpolated. The strength reduction factor is then multiplied by each of the M_n and P_n values calculated previously to determine the resulting ϕM_n and ϕP_n that define this one point on the interaction diagram.

The entire process is repeated several times, with varying levels of strain assigned to produce a series of points that define the interaction diagram boundary. *Figure 1* depicts the superimposed strain conditions as recommended by prominent textbook authors. For each level of strain, the calculations described herein are repeated. For each level of strain, a corresponding point on an interaction diagram can be determined. Interconnecting the points results in the interaction diagram (potentially similar to *Figure 3*) on which we can plot M_u, P_u and assess whether the column is sufficient.

In summary, the steps for developing a concrete column interaction diagram are:

1) Assign an arbitrary level of strain to the layer of reinforcement opposite the compressive surface and calculate the depth to the neutral axis (c), the depth of the compression stress zone (a), and the level of strain in the remaining layers of reinforcement (using linear interpolation, similar triangles, etc.).

2) Using the ultimate compressive stress in the concrete ($0.85f'_c$) and the dimensions of the compression stress zone, calculate the resultant compressive force in the concrete. Using the strains in each layer of reinforcement, calculate the stresses and the resultant forces in each layer by multiplying the stress in each layer by the area in each layer.

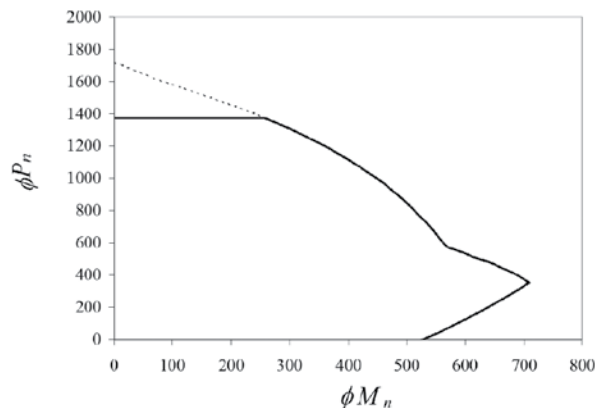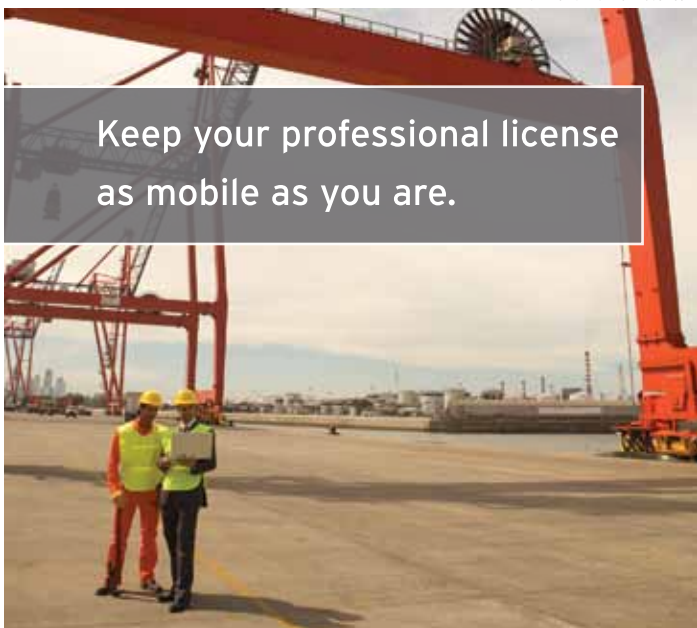3) Add the resultant concrete force and the forces in all of the layers of reinforcement to determine the nominal axial capacity (P_n). Multiply these same forces by their distances from the center of the section and add the results to yield the nominal flexural capacity (M_n). Determine the strength reduction factor and multiply it by M_n, P_n. Plot the point ϕM_n, ϕP_n on the interaction diagram.

4) Repeat steps 1-3 at least a dozen times, with strain values typically ranging from 0.5 to -6, to create enough points to draw the interaction curve. Then, simply connect the points.▪

Reinforced Concrete Column Interaction Diagram

Figure 3: Typical interaction diagram.